

ON UPPER CONDITIONALLY
COMPLETE GENERALIZED BOOLEAN LATTICES

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The article studies generalized Boolean lattices. It provides spectral characterizations of generalized Boolean lattices. The purpose of the article is to investigate properties of upper conditionally complete generalized Boolean lattices. The author gives abstract characterizations of such lattices and proves a generalization of the well-known theorem that the completeness of any Boolean lattice is equivalent to the extremally disconnection of its maximal spectrum. The work also considers upper conditionally m -complete generalized Boolean lattices for arbitrary infinite cardinality m , and provides examples and comments.

Keywords: generalized Boolean lattice, maximal spectrum, conditional completeness, extremally disconnected space, direct summand.

MSC: 06D05, 08A05

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