

**CONSTRAINED CONVEX MINIMIZATION METHODS GENERATING
REGULARIZING ALGORITHMS****V. V. Vasin, I. A. Gainova**

We investigate a modified version of the previously published method to solve the problem of minimizing a convex functional. This current modification is related to a new procedure for calculating the metric projection included in the step operator of the basic iterative process. Unlike the basic method, its modified version allows to solve the constrained convex minimization problem for compatible and incompatible system of constraints. Numerical experiments confirm the efficiency of both the basic and modified methods.

Keywords: ill-posed and improper problems, convex constraints, iterative process, convex minimization, regularizing algorithm.

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1. Vasin V.V. Fejér type iterative methods in the constrained quadratic minimization problem. *Proc. Steklov Inst. Math. (Suppl.)*, 2023, vol. 323, suppl. 1, pp. S305–S320. <https://doi.org/10.1134/S008154382306024X>
2. Vasin V.V. Stable iterative methods in problem of constrained convex minimizations. *Eur. J. Math. Comput. Appl.*, 2024, vol. 12, no. 4, pp. 150–157. <https://doi.org/10.32523/2306-6172-2024-12-4-150-157>
3. Bakushinskii A.B., Goncharskii A.V. *Iterativnyye metody resheniya nekorrektnykh zadach* [Iterative methods for solving ill-posed problems]. Moscow, Nauka Publ., 1989, 128 p. ISBN: 5-02-013960-2.
4. Vasil'ev F.P. *Chislennyye metody resheniya ekstremal'nykh zadach* [Numerical methods for solving extremal problems]. Moscow, Nauka Publ., 1980, 518 p.
5. Polyak B.T. *Introduction to optimization*. New York, Division Publ., 1987, 438 p. ISBN-10: 0911575146. Original Russian text published in Polyak B. T. *Vvedeniye v optimizatsiyu*, Moscow, Nauka Publ., 1983, 384 p.
6. Eremin I.I. Duality for improper problem of linear and convex programming. *Sov. Math. Dokl.*, 1981, vol. 23, pp. 62–66.
7. Popov L.D. Barriers and symmetric regularization of the Lagrange function in the analysis of improper linear programming problems. *Trudy Inst. Mat. Mekh. UrO RAN*, 2023, vol. 29, no. 3, pp. 138–155 (in Russian). <https://doi.org/10.21538/0134-4889-2023-29-3-138-155>
8. Skarin V.D. The method of quasi-solutions based on barrier functions in the analysis of improper convex programming problems. *Trudy Inst. Mat. Mekh. UrO RAN*, 2023, vol. 29, no. 3, pp. 168–184 (in Russian). <https://doi.org/10.21538/0134-4889-2023-29-3-168-184>
9. Halpern B. Fixed points of nonexpanding maps. *Bull. Amer. Math. Soc.*, 1967, vol. 73, no. 6, pp. 957–961. <https://doi.org/10.1090/S0002-9904-1967-11864-0>
10. Vasin V.V., Eremin I.I. *Operators and iterative processes of Fejér type. Theory and applications*, Berlin, NY, Walter de Gruyter, 2009, 155 p. ISBN: 3110218186. Original Russian text was published in Vasin V.V., Eremin I.I. *Operatory i iteratsionnye protsessy feierovskogo tipa. Teoriya i prolojeniya*, Izhevsk, Regul. Khaot. Dinamika, 2005, 200 p. ISBN: 5-93972-427-2.
11. Vasin V.V. *Osnovy teorii nekorrektnykh zadach* [Fundamentals of the theory of ill-posed problems], Novosibirsk, Publ. of Syberian Branch of Russian Acad. Sci., 2020, 312 p. ISBN: 978-5-7692-1673-2.
12. Vasin V.V., Ageev A.L. *Ill-posed problems with a priori information*, Utrecht, VSP, 1995, 255 c. ISBN: 906764191X. Original Russian text was published in Vasin V. V., Ageev A. L. *Nekorrektnye zadachi s apriornoj informatsiei*, Yekaterinburg, Ural Publ. House “Nauka”, 1993, 264 p. ISBN: 5-7691-0390-6.

13. Rockafellar R.T. Monotone operators and the proximal point algorithm. *SIAM J. Contr. Optim.*, 1976, vol. 14, no. 5, pp. 877–898. <https://doi.org/10.1137/0314056>
14. Lucchetti R., Patrone F. Hadamard and Tyhonov well-posedness of a certain class of convex functions. *J. Math. Anal. Appl.*, 1982, vol. 88, no. 1, pp. 204–215. [https://doi.org/10.1016/0022-247X\(82\)90187-1](https://doi.org/10.1016/0022-247X(82)90187-1)
15. Lawson C.L., Hansen R.J. *Solving least squares problems*. New Jersey, Englewoods Cliffs, Prentice-Hall, 1995, 337 p.

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