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AN EDGE SWITCHING PROCEDURE AND SPLITTABLE ANCESTORS OF A GRAPH

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Let r be the Durfey rank of a graph $G=(V,E)$, i.e. the Durfey rank of its degree partition. Let V_1, V_2 be a set-theoretic partition of the vertex set V into two nonempty subsets such that $\deg v \geq r$ for any $v \in V_1$ and $\deg v \geq r$ for any $v \in V_2$. The bipartite graph (V_1, E', V_2) is called the sandwich subgraph of G corresponding to the pair V_1, V_2, where E' is the set of all edges in E that go from V_1 to V_2. A graph G is splittable graph if its set of vertices can be represented as the union of a clique and a coclique. We will call a graph H a splittable ancestor of a graph G if the graph G is reducible to the graph H using some sequential lifting rotations of edges and H is a splittable graph. A splittable ancestor of a graph G of Durfey rank r is called nearest splittable r-ancestor of G if it can be obtained from G using the smallest possible number s of lifting rotations of edges. Note that $s=\frac{1}{2}(\operatorname{sum}\operatorname{tl}(\lambda)-\operatorname{sum}\operatorname{hd}(\lambda))$, where $\lambda=\operatorname{dpt}(G)$, $\operatorname{tl}(\lambda)$ is the tail and $\operatorname{hd}(\lambda)$ is the head of the partition λ. The aim of this paper is to prove the following two statements. Let r be the Durfey rank of a graph G, and let G_1 be a graph obtained from G using an edge switching procedure corresponding to some alternating 4-cycle C.

1. If C is not an alternating 4-cycle of the sandwich subgraph (V_1, E', V_2) of G, then G and G_1 have a common nearest splittable r-ancestor.

2. If C is contained in a sandwich subgraph (V_1, E', V_2) of G, then G and G_1 have a common splittable ancestor that is obtained from each of them by a sequence of at most $s+1$ lifting rotations of edges.

Keywords: integer partition, graphical partition, degree partition, splittable graph, rotation of an edge.

MSC: 05C35, 05C07

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