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ON GROUPS WHOSE CONJUGACY CLASS SIZES ARE NOT DIVISIBLE BY EACH OTHER[1]

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Let G be a finite group and $N(G)$ be the set of its conjugacy class sizes excluding 1. Let us define a directed graph $\Gamma(G)$, the set of vertices of this graph is $N(G)$ and the vertices x and y are connected by an arc from x to y if x divides y and $N(G)$ does not contain a number z different from x and y such that x divides z and z divides y. We will call the graph $\Gamma(G)$ the conjugate graph of the group G. In this work, we will study finite groups whose conjugate graph is a set of isolated vertices.

Keywords: finite group, conjugacy classes, conjugate graph.

MSC: 20D60, 20E45

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