

A NOTE ON THE NON-SOLVABLE FORMATION $\hat{\mathfrak{J}}_{pr}$ ¹

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In this paper, we extend the formation $\hat{\mathfrak{J}}_{pr}$, which is generated by the class \mathfrak{J}_{pr} originally introduced by Demina and Maslova. The class $\hat{\mathfrak{J}}_{pr}$ consists of finite groups in which every non-solvable maximal subgroup has a primary index. Building upon this framework, we introduce and study two generalized formations, denoted by $\hat{\mathfrak{J}}$ and $\hat{\mathfrak{J}}_p$, which are obtained by involving minimal non-solvable maximal subgroups and applying a localization approach to maximal subgroups. We establish new sufficient conditions under which a finite group belongs to these formations. In addition, we give examples of non-solvable groups to illustrate the distinctions between the class $\hat{\mathfrak{J}}_{pr}$ and its generalizations.

Keywords: formation, non-solvable group, second maximal subgroup, the core of subgroup.

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¹This work was supported by General Program of Natural Science Foundation of Xinjiang Uygur Autonomous Region (Grant # 2024D01C200), NSFC (Grant # 12371018), Fundamental Research Funds for the Central Universities (Grant # B240201093/2013), NSFC-RFBR (Grant # 12011530061), and Yili Normal University General Natural Science Project for Enhancing Comprehensive Discipline Strength (Grant # 22XKZY19).

Received May 22, 2025
Revised August 25, 2025
Accepted September 8, 2025

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Cite this article as: Wenxia Zhou, Long Miao, Baijun Gao, Ran Li. A note on the non-solvable formation $\hat{\mathfrak{J}}_{pr}$. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2025, vol. 31, no. 4, pp. 290–299.