

DEZA CAYLEY GRAPHS FROM DIFFERENCE SETS<sup>1</sup>

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A regular graph  $\Gamma$  is called a *Deza* graph if there exist nonnegative integers  $a$  and  $b$  such that any two distinct vertices of  $\Gamma$  have either  $a$  or  $b$  common neighbors. A subset  $R$  of a group  $G$  is called a *relative difference set* in  $G$  if there exist a subgroup  $N$  of  $G$  and a nonnegative integer  $\lambda$  such that every element of  $G \setminus N$  has exactly  $\lambda$  representations in the form  $g_1 g_2^{-1}$ , where  $g_1, g_2 \in R$ , and no non-identity element of  $N$  has such a representation. If  $N$  is trivial, then  $R$  is defined to be a *difference set*. In the present paper, we provide several new constructions of Deza Cayley graphs over groups having a generalized dihedral subgroup. These constructions are based on usage of (relative) difference sets.

Keywords: Deza graphs, Cayley graphs, difference sets.

MSC: 05B10, 20C05, 05E30

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