

ON THE  $R_\delta$ -STRUCTURE OF THE SOLUTION SET OF THE CAUCHY  
PROBLEM FOR SEMILINEAR DIFFERENTIAL INCLUSIONS  
OF FRACTIONAL ORDER  $\alpha \in (1, 2)$  IN BANACH SPACES

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In this paper, we study the topological structure of a solution set to the Cauchy problem for semilinear differential inclusions of fractional order  $\alpha(1, 2)$  in Banach spaces. It is assumed that the linear part of the inclusions is a linear closed operator generating a strongly continuous and uniformly bounded family of cosine operator functions. The nonlinear part is represented by a upper semicontinuous multivalued operator of Caratheodory type. It is established that the set of solutions to the problem is an  $R_\delta$ -set. The paper has the following structure. After the introduction, we present the necessary preliminary information from the theories of multivalued mappings, measures of noncompactness, fractional analysis, and the family of cosine operator functions. The third section is devoted to auxiliary results. In the next section, a number of lemmas and the main result of the paper (Theorem 2) are proved. In the final section, as an example of applying the obtained results, a generalized periodic problem for semilinear differential inclusions of fractional order  $\alpha \in (1, 2)$  is considered.

**Keywords:** topological structure,  $R_\delta$ -set, differential inclusion, fractional derivative, family of cosine operator functions, multivalued map, condensing multioperator.

MSC: 34G25, 34A08, 34A60

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