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ASSIMILATION OF IRREGULAR BOUNDARY DATA IN RECOVERING MODEL COEFFICIENTS

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An extremal (variational) problem for minimizing a certain residual functional is considered. The extremal problem is a variational formulation of the inverse problem of finding the thermal diffusivity in a steady-state diffusion-advection-reaction model. The initial information for solving the inverse problem is the results of measuring the trace of the normal derivative of the solution to the corresponding boundary value problem for this model at the model's operating boundary. The residual functional is the difference between the normal derivatives of the simulated and observed states of the model in the metric of negative Sobolev space at the boundary of the model's operating domain. A preliminary assertion is proved regarding the existence and uniqueness of the trace of the normal derivative of the solution in fractional-order negative Sobolev space at the boundary, allowing for a correct formulation of the inverse problem and its variational formulation. Various aspects of the extremal problem are investigated. It is shown that the set of minimum points in the variational problem may be empty. Some conditions for the solvability of a variational problem are also presented when the set of minimum points is nonempty. Some necessary conditions for the uniqueness of a minimizing element are indicated. The concepts of weak and strong well-posedness of an extremal problem are formulated. Strong well-posedness implies weak well-posedness, and some conditions for strong well-posedness are indicated. Examples of problems in which both strong and weak well-posedness of the problem are absent are given; weak but not strong well-posedness exists. Necessary conditions for the minimum of the residual functional in a special problem are formulated.

Keywords: diffusion–advection–reaction equation, thermal diffusivity coefficient, inverse problem, residual functional, extremal problem, variational method, minimum point.

MSC: 35Q30, 76D05, 76T10, 76T15

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