

VORTEX GROUP FOR KNOTS AND LINKS IN A 3-SPHERE

Ph. G. Korablev

The article is devoted to the construction of a vortex group. This group is a well defined invariant for oriented links in a 3-sphere. It is defined by using generators and relations. The generators are both the crossings of the diagram and two additional formal symbols, while the regions into which the diagram divides the 2-sphere play the role of the relations. It is proved that groups constructed for different diagrams of the same link are isomorphic. A reduced vortex group is obtained from a vortex group by trivialising two specific generators. It is proved that this group allows a balanced presentation. The construction of the reduced vortex group is close to one of the definitions of the Alexander polynomial for links. It is proved that the order of the abelianized reduced vortex group coincides with the determinant of the link.

Keywords: link, vortex group, link determinant.

MSC: 57M27, 57M25

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Philipp Glebovich Korablev, Cand. Sci. (Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia; Chelyabinsk State University, Chelyabinsk, 454001 Russia, e-mail: korablev@csu.ru.

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