

ON ESTIMATES OF n -TERM APPROXIMATIONS OF FUNCTIONS IN LORENTZ SPACE

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The article considers the Lorentz space $L_{q,\tau}(\mathbb{T}^m)$ of periodic functions of m variables and the class $W_{q,\tau}^{a,b(\cdot),\bar{F}}$ for $1 < q, \tau < \infty$, $a > 0$, $b(t)$ is a slowly varying function on $[1, \infty)$. $W_{q,\tau}^{a,b(\cdot),\bar{F}}$ the class of all functions $f \in L_{q,\tau}(\mathbb{T}^m)$ for which $S_n^{(\bar{\gamma})}(f, \bar{x})$ the partial sum over the step hyperbolic cross of the Fourier series in the norm of $L_{q,\tau}(\mathbb{T}^m)$ converges at rate $2^{-na}b(2^n)$ as $n \rightarrow \infty$. The main result is the exact order of the best n -term trigonometric approximations of functions from the class $W_{q,\tau_1}^{a,b(\cdot),\bar{F}}$ in the norm of the space $L_{p,\tau_2}(\mathbb{T}^m)$ in the case $1 < q < p \leq 2$, for some relations between the parameters a , τ_1 , τ_2 . The result is proved by a constructive method.

Keywords: Lorentz space, trigonometric system, best n -term approximation, constructive method.

MSC: 42A10, 41A46

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