

OBSERVATION CONTROL PROBLEM FOR DIFFERENTIAL EQUATIONS

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We consider a controlled linear differential equation. The controller must transfer the initial state x_0 of the equation to a given final state x_T . This process is followed by the observer, who tries to determine x_T but does not know the state vector of the equation and obtains information via the vector $y(t)$ connected with $x(t)$. With the aid of the signal $y(t)$, the observer can determine an information set containing x_T . In the case of special constraints for controls (or disturbances from the point of view of the observer), the information set becomes the ellipsoid, the parameters of which are described by the system of differential equations. In the game, the controller, who is the main player, endeavors to accomplish its task and maximize the information set simultaneously. An example is considered.

Keywords: guaranteed estimation, information set, reachable set, observation control.

MSC: 93B99

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