

METHOD OF S. B. STECHKIN AND V. T. GAVRILYUK AND ITS APPLICATION

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In 1978, S.B. Stechkin and V.T. Gavrilyuk, in their work on the rate of convergence of the Fourier series of a continuous function, found a special method for estimating the norm of deviation of a function from the partial sum of its Fourier series, using the integral properties of Dirichlet kernels. The purpose of this article is to recall the main idea of this remarkable work and to show how, using a modification of the method used in it, results were recently obtained for functions of bounded variation, bounded p -variation, an estimate of the rate of convergence in the Riemann localization principle for continuous functions was given, and other problems were solved.

Keywords: Fourier series, rate of convergence, modulus of continuity, functions of bounded variation, Riemann localization principle.

MSC: 42A10, 42A63

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