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ONE-DIMENSIONAL (k, a)-GENERALIZED FOURIER TRANSFORM

V. I. Ivanov

We study the two-parametric (k, a)-generalized Fourier transform $\mathcal{F}_{k,a}$, $k, a > 0$, on the line. For $a \neq 2$ it has deformation properties and, in particular, for a function f from the Schwartz space $\mathcal{S}(\mathbb{R})$, $\mathcal{F}_{k,a}(f)$ may be not infinitely differentiable or rapidly decreasing at infinity. It is proved that the invariant set for the generalized Fourier transform $\mathcal{F}_{k,a}$ and differential-difference operator $|x|^{2-a}\Delta_k f(x)$, where Δ_k is the Dunkl Laplacian, is the class

$$\mathcal{S}_a(\mathbb{R}) = \{f(x) = F_1(|x|^{a/2}) + xF_2(|x|^{a/2})\colon F_1, F_2 \in \mathcal{S}(\mathbb{R}),\ F_1, F_2 - \text{are even}\}.$$

For $a = 1/r$, $r \in \mathbb{N}$, we consider two generalized translation operators τ^y and $T^y = (\tau^y + \tau^{-y})/2$. Simple integral representations are proposed for them, which make it possible to prove their L^p-boundedness as $1 \leq p \leq \infty$ for $\lambda = r(2k-1) > -1/2$. For $\lambda \geq 0$ the generalized translation operator T^y is positive and its norm is equal to one. Two convolutions are defined and Young's theorem is proved for them. For generalized means defined using convolutions, a sufficient L^p-convergence condition is established. The generalized analogues of the Gauss–Weierstrass, Poisson, and Bochner–Riesz means are studied.

Keywords: (k, a)-generalized Fourier transform, generalized translation operator, convolution, generalized means.

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Valerii Ivanovich Ivanov, Dr. Phys.-Math. Sci., Prof., Tula State University, Tula, 300012 Russia, Tula State Lev Tolstoy Pedagogical University, Tula, 300026 Russia, e-mail: ivaleryi@mail.ru .

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