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KOLMOGOROV WIDTHS OF THE INTERSECTION OF TWO WEIGHTED SOBOLEV CLASSES ON AN INTERVAL WITH THE SAME SMOOTHNESS

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Order estimates are obtained for the Kolmogorov n-widths of the intersection of two weighted Sobolev classes on an interval with the same smoothness for large n. The weights have a general form, and one of them is in a certain sense significantly less than the other. The constants in the order equality are independent of the weights. Order estimates are obtained for the Kolmogorov n-widths of the intersection of two weighted Sobolev classes $W^r_{p_1,g_1}[a,\, b]$ and $W^r_{p_2,g_2}[a,\, b]$ in the weighted Lebesgue space $L_{q,v}[a,\, b]$ for large n. It is assumed that $p_1 > p_2$. The weights g_1, g_2, and v have general form. The conditions on these functions are such that the order of the width in n is the same as for the unweighted Sobolev class $W^r_{p_1}[a,\, b]$. In addition, the weight g_2 in a certain sense is considerably less than the weight g_1. The constants in the order equality for the width depend only on p_1, p_2, q, and r. The upper estimate reduces to the use of our earlier result (2010) for one weighted Sobolev class. The lower estimate is derived by using the discretization method and estimating the width of the intersection of the p_1- and p_2-ellipsoids. Then a polyhedron of special form is inscribed in this set, and the required lower estimate is obtained for the width of the polyhedron under an appropriate choice of the parameters.

Keywords: Kolmogorov widths, intersection of function classes.

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