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**SPECIAL FACTORS IN THE RESTRICTIONS OF IRREDUCIBLE MODULES  
OF CLASSICAL GROUPS TO SUBSYSTEM SUBGROUPS  
WITH TWO SIMPLE COMPONENTS****I. D. Suprunenko, T. S. Busel, A. A. Osinovskaya**

For restrictions of  $p$ -restricted irreducible modules of classical algebraic groups in odd characteristic  $p$  with highest weights that are relatively large with respect to  $p$  to a subsystem subgroup  $H$  of maximal rank with two main components  $H_1$  and  $H_2$  under slight constraints restrictions on the ranks of the subgroups  $H_1$  and  $H_2$ , a lower bound is found for the number of composition factors that are  $p$ -large for the subgroup  $H_1$  and not too small for  $H_2$ ; the bound grows as the highest weight increases. On this basis, lower bounds are obtained for the number of Jordan blocks of maximal size for the images of certain unipotent elements in the corresponding representations of the groups.

Keywords: classical algebraic groups, modular representations, restrictions, composition factors, unipotent elements, Jordan blocks.

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