

MSC: 20B07, 20F50

DOI: 10.21538/0134-4889-2023-29-4-241-258

AT-GROUPS

A. V. Rozhkov, V. Yu. Barsukova

Periodic nonlocally finite (Burnside) groups of infinite period are studied. The first explicitly given example of such a group was proposed by S. V. Aleshin in 1972. His construction was generalized to AT-groups, which are automorphism groups of trees. A number of well-known problems have been solved with the help of AT-groups. This work is a continuation and development of the previous article by one of the authors. A new strategy for studying AT-groups has been implemented. The examples of Alyoshin, Sushanskii, and Gupta, which have already become classical, but, as it turned out, are poorly studied, are reviewed again. A well-studied example of Grigorchuk's 2-group is generalized and reviewed in a new way. New classes of AT-groups are introduced. Tasks for the hour of problems are proposed.

Keywords: Burnside groups, residually finite groups, finiteness conditions, AT-groups, trees, wreath products.

REFERENCES

1. Golod E.S. On nil-algebras and finitely approximable p -groups. *Amer. Math. Soc., Translat., II*, 1965, vol. 48, pp. 103–106. doi: 10.1090/trans2/048
2. Aleshin S.V. Finite automata and Burnside's problem for periodic groups. *Math. Notes*, 1972, vol. 11, no. 3, pp. 199–203. doi: 10.1007/BF01098526
3. Sushchanskii V.I. Periodic p -groups of permutations and the unrestricted Burnside problem. *Soviet Math. Dokl.*, 1979, vol. 20, pp. 766–770.
4. Grigorchuk R.I. Burnside problem on periodic groups. *Funct. Anal. Appl.*, 1980, vol. 14, no. 1, pp. 41–43. doi: 10.1007/BF01078416
5. Gupta N., Sidki S. Some infinite p -groups. *Algebra and Logic*, 1983, vol. 22, no. 5, pp. 421–424. doi: 10.1007/BF01982120
6. Rozhkov A.V. On subgroups of infinite finitely generated p -groups. *Math. USSR-Sbornik*, 1987, vol. 57, no. 2, pp. 437–448. doi: 10.1070/SM1987v057n02ABEH003078
7. Grigorchuk R.I. Degrees of growth of finitely generated groups and the invariant mean. *Izvestiya Akad. Nauk SSSR, Ser. Matem.*, 1984, vol. 48, no. 5, pp. 572–589 (in Russian).
8. Rozhkov A.V. Conditions of finiteness in groups of automorphisms of trees: Diss. ... Dr. Phys.-Math. Sci. Krasnoyarsk, Krasnoyarsk State Univ., 1997, 230 p. (in Russian)
9. *The Kourovka notebook. Unsolved problems in group theory, 20th ed.*, eds. V.D. Mazurov, E.I. Khukhro, Novosibirsk: Inst. Math. SO RAN Publ., 2022, 269 p. Available at: <https://kourovka-notebook.org/>.
10. Rozhkov A.V. AT-groups which are not AT-subgroups: transition from AT_ω -groups to AT_Ω -groups. *Trudy Inst. Mat. Mekh. UrO RAN*, 2022, vol. 28, no. 1, pp. 218–231 (in Russian). doi: 10.21538/0134-4889-2022-28-1-218-231
11. Merzlyakov Yu.I. On infinite finitely generated periodic groups. *Soviet Math. Dokl.*, 1983, vol. 27, pp. 169–172.
12. Pervova E.L. The congruence property of AT-groups. *Algebra and Logic*, 2002, vol. 41, no. 5, pp. 306–313. doi: 10.1023/A:1020979720331
13. Ol'shanskii A.Yu. *The geometry of defining relations in groups*, Dordrecht, Kluwer Publ., 1991, 505 p. ISBN: 9780792313946. Original Russian text was published in Ol'shanskii A. Yu., *Geometriya opredelyayushchikh sootnoshenii v gruppakh*, Moscow, Nauka Publ., 1989, 448 p. ISBN: 5-02-013916-5.

14. Kargapolov M.I., Merzljakov Ju.I. *Fundamentals of the theory of groups*. Transl. from the 2nd Russian ed. Ser. Graduate Texts in Mathematics, vol. 62. NY; Heidelberg; Berlin: Springer-Verlag. 1979, 203 p. ISBN: 978-1-4612-9966-0 . Original Russian text (3rd ed.) published in Kargapolov M.I., Merzlyakov Yu.I. *Osnovy teorii grupp*, Moscow, Nauka Publ., 1982, 288 p.
15. Sozutov A.I. On some infinite groups with a strongly embedded subgroup. *Algebra and Logic*, 2000, vol. 39, no. 5, pp. 345–353. doi: 10.1007/BF02681619
16. Lysenok I.G. A system of defining relations for a Grigorchuk group. *Math. Notes*, 1985, vol. 38, no. 4, pp. 784–792. doi: 10.1007/BF01158402
17. Grigorchuk R.I. Branch groups. *Math. Notes*, 2000, vol. 67, no. 6, pp. 718–723. doi: 10.1007/BF02675625

Received September 24, 2023

Revised November 12, 2023

Accepted November 20, 2023

Alexander Viktorovich Rozhkov, Dr. Phys.-Math. Sci., Prof., Kuban State University, Krasnodar, 350040 Russia, e-mail: ros@math.kubsu.ru .

Victoria Yurievna Barsukova, Cand. Sci. (Phys.-Math.), Kuban State University, Krasnodar, 350040 Russia, e-mail: barsukova.v.y@gmail.com .

Cite this article as: A. V. Rozhkov, V. Yu. Barsukova. AT-groups. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2023, vol. 29, no. 4, pp. 241–258 .