Trudy Instituta Matematiki i Mekhaniki UrO RAN

Vol. 29 No. 4 2023

MSC: 26A51, 26D07, 49K35

DOI: 10.21538/0134-4889-2023-29-4-193-216

ON THE WEIGHTED TRIGONOMETRIC BOJANOV–CHEBYSHEV EXTREMAL PROBLEM

Béla Nagy and Szilárd Gy. Révész

We investigate the weighted Bojanov–Chebyshev extremal problem for trigonometric polynomials, that is, the minimax problem of minimizing $\|T\|_{w,C(\mathbb{T})}$, where w is a sufficiently nonvanishing, upper bounded, nonnegative weight function, the norm is the corresponding weighted maximum norm on the torus $\mathbb{T}$, and T is a trigonometric polynomial with prescribed multiplicities $\nu_1, \ldots, \nu_n$ of root factors $|\sin(\pi(t-z_j))|^{\nu_j}$. If the ν_j are natural numbers and their sum is even, then T is indeed a trigonometric polynomial and the case when all the ν_j are 1 covers the Chebyshev extremal problem. Our result will be more general, allowing, in particular, so-called generalized trigonometric polynomials. To reach our goal, we invoke Fenton's sum of translates method. However, altering from the earlier described cases without weight or on the interval, here we find different situations, and can state less about the solutions.

Keywords: minimax and maximin problems, kernel function, sum of translates function, vector of local maxima, equioscillation, majorization.

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Received August 24, 2023
Revised October 18, 2023
Accepted November 6, 2023

Funding Agency: This research of Béla Nagy was supported by project TKP2021-NVA-09. Project no. TKP2021-NVA-09 has been implemented with the support provided by the Ministry of Innovation and Technology of Hungary from the National Research, Development and Innovation Fund, financed under the TKP2021-NVA funding scheme. The work of Sz. Gy. Révész was supported in part by Hungarian National Research, Development and Innovation Fund project # K-132097.

Béla Nagy, Department of Analysis, Bolyai Institute, University of Szeged Aradi vértanuk tere 1, 6720 Szeged, Hungary, e-mail: nbela@math.u-szeged.hu .

Szilárd Gy. Révész, HUN-REN Alfréd Rényi Institute of Mathematics, Reáltanoda utca 13-15, 1053 Budapest, Hungary, e-mail: revesz.szilard@renyi.hu .

Cite this article as: Béla Nagy and Szilárd Gy. Révész. On the weighted trigonometric Bojanov–Chebyshev extremal problem. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2023, vol. 29, no. 4, pp. 193–216 .