Trudy Instituta Matematiki i Mekhaniki UrO RAN

Vol. 29 No. 4 2023

MSC: 20D40

DOI: 10.21538/0134-4889-2023-29-4-181-192

ARITHMETIC GRAPHS AND FACTORIZED FINITE GROUPS

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The Hawkes graph $\Gamma_H(G)$ of a group G is the directed graph with vertex set $\pi(G)$ that has an edge (p, q) whenever $q \in \pi(G/O_{p',p}(G))$. The Sylow graph $\Gamma_s(G)$ of a group G is the directed graph with vertex set $\pi(G)$ that has an edge (p, q) whenever $q \in \pi(N_G(P)/PC_G(P))$ for some Sylow p-subgroup P of G. The N-critical graph $\Gamma_{Nc}(G)$ of a group G is the directed graph with vertex set $\pi(G)$ that has an edge (p, q) whenever G contains a Schmidt (p, q)-subgroup, i.e., a Schmidt $\{p, q\}$-subgroup with a normal Sylow p-subgroup. The paper studies the Hawkes, Sylow, and N-critical graphs of products of totally permutable, mutually permutable, and $\mathfrak{N}$-connected subgroups.

Keywords: finite group, Hawkes graph, Sylow graph, N-critical graph, product of totally permutable subgroups, product of mutually permutable subgroups, $\mathfrak{N}$-connected subgroups.

REFERENCES

1. Abe S., Iiyori N. A generalization of prime graphs of finite groups. *Hokkaido Math. J.*, 2000, vol. 29, pp. 391–407. doi: 10.14492/hokmj/1350912979
2. Cayley A. Desiderata and suggestions. 2. the theory of groups: graphical representation. *Amer. J. Math.*, 1878, vol. 1, no. 2, pp. 174–176. doi: 10.2307/2369306
3. Erwin D., Russo F.G. The influence of the complete nonexterior square graph on some infinite groups. *Lith. Math. J.*, 2016, vol. 56, no. 4, pp. 492–502. doi: 10.1007/s10986-016-9331-2
4. Hawkes T. On the class of Sylow tower groups. *Math. Z.*, 1968, vol. 105, no. 5, pp. 393–398. doi: 10.1007/BF01110301
5. Kazarin L.S., Martínez-Pastor A, Pérez-Ramos M.D. On the Sylow graph of a group and Sylow normalizers. *Israel J. Math.*, 2011, vol. 186, no. 1, pp. 251–271. doi: 10.1007/s11856-011-0138-x
6. Kondrat'ev A.S. Prime graph components of finite simple groups. *Mathematics of the USSR-Sbornik*, 1990, vol. 67, no. 1, pp. 235–247. doi: 10.1070/SM1990v067n01ABEH001363
7. Lucchini A., Maróti A. On the clique number of the generating graph of a finite group. *Proc. Amer. Math. Soc.*, 2009, vol. 137, pp. 3207–3217. doi: 10.1090/S0002-9939-09-09992-4
8. Russo F.G. Problems of connectivity between the Sylow graph, the prime graph and the non-commuting graph of a group. *Adv. Pure Math.*, 2012, vol. 2, pp. 391–396. doi: 10.4236/apm.2012.26058
9. Vasilyev A.F., Murashka V.I. Arithmetic graphs and classes of finite groups. *Sib. Math. J.*, 2019, vol. 60, no. 1, pp. 41–55. doi: 10.1134/S0037446619010051
10. Williams J.S. Prime graph components of finite groups. *J. Algebra*, 1981, vol. 69, no. 2, pp. 487–513. doi: 10.1016/0021-8693(81)90218-0
11. Vasil'ev A.F., Murashka V.I., Furs A.K. On the Hawkes graphs of finite groups. *Sib. Math. J.*, 2022, vol. 63, no. 5, pp. 849–861. doi: 10.1134/S0037446622050044
12. D'Aniello A., De Vivo C., Giordano G. Lattice Formations and Sylow Normalizers: A Conjecture. *Atti Semin. Mat. Fis. Univ. Modena.*, 2007, vol. 55, pp. 107–112.
13. Murashka V.I. On the connected components of the prime and Sylow graphs of a finite group. *Arch. Math.*, 2022, vol. 118, no. 3, pp. 225–229. doi: 10.1007/s00013-021-01694-x
14. Murashka V.I. N-critical graph of finite groups. *Asian-European J. Math.*, 2021, vol. 15, no. 9, 2250163. doi: 10.1142/S1793557122501637
15. Carocca A. A note on the product of F-subgroups in a finite group. *Proc. Edinburgh Math. Soc.*, 1996, vol. 39, no, 1, pp. 37–42. doi: 10.1017/S0013091500022756

16. Francalanci G. Nilpotence relations in products of groups. *J. Group Theory*, 2021, vol. 24, no. 3, pp. 467–480. doi: 10.1515/jgth-2020-0135
17. Hauck P., Martínez-Pastor A., Pérez-Ramos M.D. Products of $\mathcal{N}$-connected groups. *Illinois J. Math.*, 2003, vol. 47, no. 4, pp. 1033–1045. doi: 10.1215/ijm/1258138089
18. Ballester-Bolinches A., Esteban-Romero R., Asaad M. *Products of finite groups*. Ser. De Gruyter Expos. Math., vol. 53, Berlin; NY: Walter De Gruyter, 2010. doi: 10.1515/9783110220612
19. Vasil'ev A.F. On the problem of the enumeration of local formations with a given property. *Questions in Algebra*, 1987, no. 3, pp. 3–11 (in Russian).
20. Ballester-Bolinches A. Ezquerro L.M. *Classes of finite groups*. Ser. Math. Appl., vol. 584, Netherlands: Springer, 2006, 359 p.
21. Beidleman J.C., Heineken H. Mutually permutable subgroups and group classes. *Arch. Math.*, 2005, vol. 85, no. 1, pp. 18–30. doi: 10.1007/s00013-005-1200-2
22. Vasil'ev A.F., Vasil'eva T.I., Simonenko D.N. On MP-closed saturated formations of finite groups. *Russian Math.*, 2017, vol. 61, no. 6, pp. 6–12. doi: 10.3103/S1066369X17060020
23. Shemetkov L.A., Skiba A.N. *Formations of algebraic systems*. Moscow: Nauka Publ., 1989, 256 p. (in Russian)
24. Aivazidis S., Safonova I.N., Skiba A.N. Subnormality and residuals for saturated formations: A generalization of Schenkman's theorem. *J. Group Theory*, 2021, vol. 24, no. 4, pp. 807–818. doi: 10.1515/jgth-2020-0149
25. Murashka V.I. Groups with prescribed systems of Schmidt subgroups. *Sib. Math. J.*, vol. 60, no. 2, pp. 334–342. doi: 10.1134/S0037446619020149

Received June 9, 2023
Revised August 8, 2023
Accepted August 28, 2023

Funding Agency: This work was supported by the Belarusian Republican Foundation for Fundamental Research (project no. Ф23РНФ-237).

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Cite this article as: V. I. Murashka. Arithmetic graphs and factorized finite groups. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2023, vol. 29, no. 4, pp. 181–192.