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ON SUBMODULARITY AND K$\mathfrak{F}$-SUBNORMALITY IN FINITE GROUPS

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Let $\mathfrak{F}$ be a formation, and let G be a finite group. A subgroup H of G is K$\mathfrak{F}$-subnormal (submodular) in G if there is a subgroup chain $H = H_0 \leq H_1 \leq \ldots \leq H_{n-1} \leq H_n = G$ such that for every i either H_i is normal in H_{i+1} or $H_{i+1}^{\mathfrak{F}} \leq H_i$ (H_i is a modular subgroup of H_{i+1}, respectively). We prove that in a group, a primary subgroup is submodular if and only if it is K$\mathfrak{U}_1$-subnormal. Here $\mathfrak{U}_1$ is a formation of all supersolvable groups with square-free orders of elements. Moreover, for a solvable subgroup-closed formation $\mathfrak{F}$, every solvable K$\mathfrak{F}$-subnormal subgroup of a group G is contained in the solvable radical of G. We also obtain a series of applications of these results to the investigation of groups factorized by K$\mathfrak{F}$-subnormal and submodular subgroups.

Keywords: finite group, subnormal subgroup, submodular subgroup.

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