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ON PERIODIC GROUPS WITH A FINITE NONTRIVIAL SYLOW 2-SUBGROUP

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The following results are proved. Let d be a natural number, and let G be a group of finite even exponent such that each of its finite subgroups is contained in a subgroup isomorphic to a direct product of m dihedral groups, where $m \leqslant d$. Then G is finite (and isomorphic to a direct product of at most d dihedral groups). Next, suppose that G is a periodic group and p is an odd prime. If every finite subgroup of G is contained in a subgroup isomorphic to a direct product $D_1 \times D_2$, where D_i is a dihedral group of order $2p^{r_i}$ with natural r_i, $i = 1, 2$, then $G = M_1 \times M_2$, where $M_i = \langle H_i, t\rangle$, t_i is an element of order 2, H_i is a locally cyclic p-group, and $h^{t_i} = h^{-1}$ for every $h \in H_i$, $i = 1, 2$. Now, suppose that d is a natural number and G is a solvable periodic group such that every of its finite subgroups is contained in a subgroup isomorphic to a direct product of at most d dihedral groups. Then G is locally finite and is an extension of an abelian normal subgroup by an elementary abelian 2-subgroup of order at most 2^{2d}.

Keywords: periodic group, exponent, Sylow 2-subgroup, dihedral group, direct product, saturating set.

REFERENCES

1. Ivanov S. V. The free Burnside groups of sufficiently large exponents. *Internat. J. Algebra Comput.*, 1994, vol. 4, no. 1–2, pp. 1–308. doi: 10.1142/S0218196794000026
2. Lysenok I. G. Infinite Burnside groups of even exponent. *Izv. Math.*, 1996, vol. 60, no. 3, pp. 453–654. doi: 10.1070/IM1996v060n03ABEH000077
3. Ivanov S. V., Olshanskii A. Yu. On finite and locally finite subgroups of free Burnside groups of large even exponents. *J. Algebra*, 1997, vol. 195, no. 1, pp. 241–284. doi: 10.1006/jabr.1996.6941
4. Shlepkin A. K., Rubashkin A. G. A class of periodic groups. *Algebra and Logic*, 2005, vol. 44, no. 1, pp. 65–71. doi: 10.1007/s10469-005-0008-x
5. Shlepkin A. K. On certain torsion groups saturated with finite simple groups. *Siberian Adv. Math.*, 1999, vol. 9, no. 2, pp. 100–108.
6. Amberg B., Kazarin L. Periodic groups saturated by dihedral subgroups. In: *Proc. Conf. Ischia group theory 2010*, World Sci. Publ., 2012, pp. 11–19. doi: 10.1142/9789814350051_0002
7. Belousov I. N., Kondrat'ev A. S., Rozhkov A. V. The 12th school–conference on group theory dedicated to the 65th birthday of A.A. Makhnev (informational paper). *Trudy Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 3, pp. 286–295 (in Russian). doi: 10.21538/0134-4889-2018-24-3-286-295
8. Kukharev A. V., Shlepkin A. A. Locally finite groups saturated with direct product of two finite dihedral groups. *Izvestiya Irkutsk. Gos. Univ., Ser. Matem.*, 2023, vol. 44, pp. 71–81 (in Russian). doi: 10.26516/1997-7670.2023.44.71
9. Shunkov V. P. On periodic groups with almost regular involution. *Algebra i Logika*, 1972, vol. 11, no. 4, pp. 470–493 (in Russian).
10. Lytkina D. V., Tukhvatullina L. R., Filippov K. A. The periodic groups saturated by finitely many finite simple groups. *Siberian Math. J.*, 2008, vol. 49, no. 2, pp. 317–321. doi: 10.1007/s11202-008-0031-y
11. Lytkina D. V., Mazurov V. D. Fusion of 2-elements in periodic groups with finite Sylow 2-subgroups. *Sib. Electron. Math. Reports*, 2020, vol. 17, pp. 1953–1958. doi: 10.33048/semi.2020.17.131
12. Hall M. *The theory of groups*, NY, Macmillan, 1959, 434 p. Translated to Russian under the title *Teoriya grupp*, Moscow, Inostr. Liter. Publ., 1962, 467 p.

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