EXAMPLES OF NONPRONORMAL RELATIVELY MAXIMAL SUBGROUPS IN FINITE SIMPLE GROUPS

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Using R. Wilson's recent results, we prove the existence of triples $(\mathfrak{X}, G, H)$ such that $\mathfrak{X}$ is a complete (i.e. closed under taking subgroups, homomorphic images, and extensions) class of finite groups, G is a finite simple group, H is an $\mathfrak{X}$-maximal subgroup of G, and H is not pronormal in G. This disproves a conjecture stated earlier by the second author and W. Guo.

Keywords: complete class of groups, relatively maximal subgroup, pronormal subgroup, finite simple group.

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