NIKOL'SKII'S INEQUALITY OF DIFFERENT METRICS FOR TRIGONOMETRIC POLYNOMIALS IN A SPACE WITH MIXED ASYMMETRIC NORM

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A Lebesgue space of 2π-periodic functions of m variables with a mixed norm is considered. Based on this Lebesgue space, a space with a mixed asymmetric norm is defined. The main aim of the paper is to prove Nikol'skii's inequality of different metrics for multiple trigonometric polynomials in spaces with mixed asymmetric norms. The paper consists of an introduction and three sections. In the first section, several auxiliary statements about the asymmetric norm of a multiple trigonometric polynomial are proved. In the second section, Nikol'skii's inequality of different metrics is proved for multiple trigonometric polynomials in spaces with mixed asymmetric norms. In the third section, the accuracy of Nikol'skii's inequality for multiple trigonometric polynomials is established. An extremal polynomial is constructed.

Keywords: space with asymmetric norm, Nikol'skii's inequality of different metrics, trigonometric polynomial.

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