

MSC: 91A06, 91A10

DOI: 10.21538/0134-4889-2021-27-3-71-86

ON ONE HYBRID EQUILIBRIUM

V. I. Zhukovskiy, K. N. Kudryavtsev

The notion of BN -hybrid equilibrium is proposed for a noncooperative N -person game. It is assumed that each player belongs to one of two classes: altruists and pragmatists. The altruists and the pragmatists choose their strategies using the concepts of the Berge equilibrium and the Nash equilibrium, respectively. Using a specially constructed Germeier convolution based on payoff functions, we obtain sufficient conditions for the existence of a BN -hybrid equilibrium. For an extension of the game with mixed strategies, a theorem on the existence of a BN -hybrid equilibrium is proved under constraints standard for mathematical game theory, namely, under the assumptions that the sets of the players' strategies are convex and compact and their payoff functions are continuous. The proposed concept is extended to noncooperative N -person games under interval uncertainty. An existence theorem is given for a strongly guaranteed N -hybrid equilibrium in mixed strategies.

Keywords: Nash equilibrium, Berge equilibrium, uncertainty, Germeier convolution.

REFERENCES

1. Nash J. Equilibrium points in N -person games. *Proc. Nat. Acad. Sci. USA*, 1950, vol. 36, no. 1, pp. 48–49. doi: 10.1073/pnas.36.1.48.
2. Nash J. Non-cooperative games. *Ann. Mathematics*, 1951, vol. 54, no. 2, pp. 286–295. doi: 10.2307/1969529.
3. Zhukovskiy V.I. Some problems of non-antagonistic differential games. In: *Mathematical Methods in Operations Research (ed. P. Kenderov)*, Sofia: Bulgaria Academy of Sciences, 1985, pp. 103–195.
4. Berge C. *Théorie générale des jeux à n personnes*. Paris: Gauthier-Villar, 1957, 114 p.
5. Vaisman K.S. *The Berge equilibrium*. THESIS: PhD thesis in Russian. Publisher: St. Petersburg University, St. Petersburg, Russia.
6. Vaisman K.S. Berge equilibrium in one differential game. In: *Slozhnye dinamicheskie sistemy: sbornik nauchnykh trudov*. Pskov: Pskovskii Pedagogicheskii Institut Publ., 1994, pp. 58–63 (in Russian).
7. Vaisman K.S. The existence of a guaranteed Berge equilibrium in one differential game. In: *Pontryagin Readings - VI: Abstract*. Voronezh, 1995, p. 19 (in Russian).
8. Vaisman K.S., Aimukhanov N.Zh. Berge equilibrium in differential-difference game. In: *Sophisticated Controlled Systems: Interuniversity Collection of Scientific Papers*. Moscow: RosZITLP, 1996, pp. 90–93 (in Russian). ISBN: 5-85507-077-8.
9. Vaisman K.S., Zhukovskiy V.I. Properties of Berge equilibrium. In: *Abstract of Intern. Workshop "Mathematical Problems of Ecology"*, Chita, 1994, pp. 27–28 (in Russian).
10. Vaisman K.S., Zhukovskiy V.I. The structure of Berge equilibrium solutions. In: *Pontryagin Readings - V: Abstract*, Voronezh, 1994, p. 29 (in Russian).
11. Zhukovskiy V.I., Vaisman K.S. About one solution in non-cooperative game. In: *Game Theory and Economics: Abstract of N.N. Vorob'ev Memorial Conference*. St.-Petersburg, 1996, p. 77.
12. Guseinov A.A., Zhukovskiy V.I., Kudryavtsev K.N. *Matematicheskie osnovy Zolotogo pravila: Teoriya novogo, al'truisticheskogo uravnoveshivaniya konfliktov v protivopolozhnost' "egoisticheskomu" ravnovesiyu po Neshu* [Mathematical foundations of the Golden Rule: the altruistic way of the conflict solution as opposed to the selfish Nash equilibrium]. Moscow: URSS, 2016, 280 p. ISBN: 978-5-9710-3213-7.
13. Bryant J. A simple rational expectations Keynes type model. *Quarterly J. Economics*, 1983, vol. 98, no. 3, pp. 525–529. doi: 10.2307/1886025.

14. Zhukovskiy, V.I., Kudryavtsev, K.N. Pareto-optimal Nash equilibrium: Sufficient conditions and existence in mixed strategies. *Autom. Remote Control*, 2016, vol. 77, pp. 1500–1510. doi: 10.1134/S0005117916080154.
15. Zhukovskiy, V.I., Kudryavtsev, K.N. Mathematical foundations of the Golden Rule. I. Static case. *Autom. Remote Control*, 2017, vol. 78, pp. 1920–1940. doi: 10.1134/S0005117917100149.
16. Zhukovskiy V.I., Zhukovskaya L.V., Kudryavtsev K.N., Larbani M. Strong coalitional equilibria in games under uncertainty. *Vestn. Udmurtsk. Univ. Mat. Mekh. Komp. Nauki*, 2020, vol. 30, no. 2, pp. 189–207. doi: 10.35634/vm200204.
17. Germeier Yu.B. *Vvedenie v teoriyu issledovaniya operatsii* [Introduction to the operations research theory]. Moscow: Nauka Publ., 1971, 383 p.
18. Germeier Yu.B. *Igry s neprotivopolozhnyimi interesami* [Games with non-antagonistic interests]. Moscow: Nauka Publ., 1976, 327 p.
19. Morozov V.V., Sukharev A.G., Fedorov V.V. *Issledovanie operatsii v zadachakh i uprazhneniyakh* [Research of the operations in tasks and exercises]. Moscow: Nauka Publ., 1986, 285 p.
20. Borel E. The theory of play and integral equations with skew symmetric kernels. *Econometrica*, 1953, vol. 21, no. 1, pp. 97–100. doi: 10.2307/1906946.
21. Von Neumann J. Zur theorie der gesellschaftsspiele. *Math. Ann.*, 1928, vol. 100, no. 1, pp. 295–320. doi: 10.1007/BF01448847.
22. Lusternik L.A. Sobolev V.J. *Elements of functional analysis*. (International monographs on advanced mathematics and physics). Delhi: Hindustan Publishing Corp., 1974, 360 p. ISBN: 0470556501. Original Russian text published in Lyusternik L.A., Sobolev V.I. *Elementy funktsional'nogo analiza*. Moscow: Nauka Publ., 1965, 520 p.
23. Krasovskii N.N., Subbotin A.I. *Game-theoretical control problems*. New York: Springer, 1988, 517 p. ISBN: 978-1-4612-8318-8. Original Russian text published in Krasovskii N.N., Subbotin A.I. *Pozitsionnye differentsial'nye igry*. Moscow: Nauka Publ., 1974, 456 p.
24. Dmitruk A.V. *Vypuklyi analiz: Elementarnyi vvodnyi kurs* [Convex analysis: Elementary introductory course]. Moscow: MAKS-PRESS, 2012, 172 p. ISBN: 978-5-89407-472-6.
25. Glicksberg I.L. A further generalization of the Kakutani fixed point theorem, with application to Nash equilibrium points. *Proc. Amer. Math. Soc.*, 1952, vol. 3, no. 1, pp. 170–174. doi: 10.2307/2032478.
26. Zhukovskiy V.I., Kudryavtsev K.N. Equilibrating conflicts under uncertainty. II. Analogue of a maximin. *Mat. Teor. Igr Pril.*, 2013, vol. 5, no. 2, pp. 3–45 (in Russian).

Received April 21, 2021

Revised May 28, 2021

Accepted June 21, 2021

Funding Agency: This work was supported jointly by the Russian Foundation for Basic Research and Chelyabinsk Oblast (project no. 20-41-740027).

Vladislav Iosifovich Zhukovskiy, Dr. Phys.-Math. Sci., Prof., Lomonosov Moscow State University, Moscow, 119991 Russia, e-mail: zhkvlad@yandex.ru.

Konstantin Nikolaevich Kudryavtsev, Cand. Sci. (Phys.-Math.), South Ural State University, Chelyabinsk, 454080 Russia, e-mail: kudrkn@gmail.com.

Cite this article as: V. I. Zhukovskiy, K. N. Kudryavtsev. On one hybrid equilibrium, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2021, vol. 27, no. 3, pp. 71–86.