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## ON THE METHOD OF PENALTY FUNCTIONS FOR CONTROL SYSTEMS WITH STATE CONSTRAINTS UNDER INTEGRAL CONSTRAINTS ON THE CONTROL

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We consider a nonlinear control system with state constraints. The system is linear in the control variables, and the control constraints are given by a quadratic integral inequality. A procedure for eliminating the state constraints is proposed for the approximate construction of the reachable set. The procedure is based on introducing an auxiliary unconstrained control system whose right-hand side depends on a small parameter. Under certain conditions on the behavior of the velocities of the system at the boundary of the state constraints, we prove the convergence of the reachable sets of the auxiliary system to the reachable set of the original system in the Hausdorff metric as the small parameter tends to zero. The results of numerical simulation are presented.

Keywords: control system, integral constraints, reachable set, state constraints.

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