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A SURVEY OF HOPF–LAX FORMULAS AND QUASICONVEXITY IN PDES

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This is a short survey of recent results obtained by the author and collaborators primarily on Hopf–Lax formulas for Hamilton–Jacobi equations and obstacle problems. The initiation of the use of quasiconvex (i.e., level convex) functions in L^∞ control and differential games led to such formulas and is briefly reviewed. Dedicated to the memory of Academician A. I. Subbotin.

Keywords: Hopf–Lax, viscosity solution, Hamilton–Jacobi, quasiconvex.

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