

MSC: 49N70

DOI: 10.21538/0134-4889-2021-27-3-227-236

NOTES ON DIFFERENTIAL GAMES

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Some problems of the theory of nonzero-sum differential games are discussed. An example of a coalition-free differential two-person game with infinitely many outcomes generated by Nash ε -equilibrium as $\varepsilon \rightarrow 0$ in the class of recursive strategies is presented in the first section. Moreover, for this example, two equilibrium situations are found in the class of program strategies such that the corresponding outcomes do not dominate each other, while all the other outcomes are dominated by at least one of these two outcomes. Thus, it is proved that in this game the same problem is present that was revealed earlier for the known bimatrix game Battle of the Sexes. In the general case, all this emphasizes the impossibility of the correct definition of the value function in a coalition-free differential game. The so-called dynamic stability problem in the theory of cooperative differential games is discussed in the second section. In particular, an example disproving the known statement on the dynamic stability (or, in other words, time consistency) of the Pareto optimality principle is given.

Keywords: nonzero-sum differential games, Nash equilibrium, Nash ε -equilibrium, Pareto optimality principle, dynamic instability.

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Received March 26, 2021

Revised May 21, 2021

Accepted June 15, 2021

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Cite this article as: S. V. Chistyakov. Notes on differential games, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2021, vol. 27, no. 3, pp. 227–236.