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PROBABILISTIC SOLUTIONS OF CONDITIONAL OPTIMIZATION PROBLEMS

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Optimization problems with random parameters are studied. The traditional approach to their solution consists in finding a deterministic solution satisfying a certain criterion: optimization of the expected value of the objective function, optimization of the probability of attaining a certain level, or optimization of the quantile. In this review paper, we consider a solution of a stochastic optimization problem in the form of a random vector (or a random set). This is a relatively new class of problems, which is called "probabilistic optimization problems." It is noted that the application of probabilistic solutions in problems with random parameters is justified in the cases of multiple decision makers. Probabilistic optimization problems arise, for example, in the analysis of multicriteria problems; in this case, the weight coefficients of the importance of criteria are regarded as a random vector. We consider important examples of economic–mathematical models, which are optimization problems with a large number of decision makers: the problem of optimal choice based on the consumer's preference function, the route selection problem based on the optimization of the generalized cost of the trip, and the securities portfolio problem with a distribution of the investors' risk tolerance. Mathematical statements of these problems are given in the form of problems of probabilistic optimization. Some properties of the constructed models are studied; in particular, the expected value of the probabilistic solution of an optimization problem is analyzed.

Keywords: probabilistic optimization, stochastic optimization, probabilistic solution, multicriteria optimization, linear convolution of criteria, consumer choice, preference function, route selection, securities portfolio problem.

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