ON ASYMPTOTIC OPTIMIZATION METHODS
FOR QUASILINEAR CONTROL SYSTEMS

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Mathematical models of dynamical systems containing small parameters in nonlinearities are usually called quasilinear systems. We present a survey of results obtained for problems of optimization of quasilinear dynamical systems in the Minsk scientific school on optimal control. We consider time-optimal control problems, terminal control problems with variable right ends of trajectories, minimum force control problems, and problems of minimization of integral quadratic functionals. The research is based on the idea of a special finite-dimensional parameterization of optimal controls. The computation of asymptotic approximations to optimal controls in the quasilinear problems under consideration is reduced to solving some basic problems, which, unlike the original problems for quasilinear systems, are optimization problems for linear systems, to the integration of linear differential equations, and to finding roots of nonsingular linear algebraic systems.

Keywords: quasilinear systems, small parameter, asymptotic approximation, finite-dimensional parameterization, optimal control, feedback control.

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