TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 24 No. 4 **2018**

MSC: 41A45, 42B05, 35S05, 58J40

DOI: 10.21538/0134-4889-2018-24-4-57-79

LINEAR RECOVERY OF PSEUDODIFFERENTIAL OPERATORS ON CLASSES OF SMOOTH FUNCTIONS ON AN m-dIMENSIONAL TORUS. I

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We construct a linear method for the recovery of pseudodifferential operators on an m-dimensional torus with symbols from particular classes with the use of linear spectral information on the symbol of the operator and on the function (finite sets of their Fourier coefficients). Error bounds are given for the error of recovery in the space $L_r(\mathbb{T}^m)$ of values of these pseudodifferential operators on elements of Nikol'skii–Besov and Lizorkin–Triebel function spaces for a number of relations between r and the parameters of the symbol classes and the function spaces (Theorem 1). A key role in the proof of the bounds is played by the boundedness of the pseudodifferential operators between appropriate Nikol'skii–Besov (Lizorkin–Triebel) function spaces (Theorem 2).

Keywords: pseudodifferential operator on m-dimensional torus, class of symbols (of product type), Nikol'skii–Besov / Lizorkin–Triebel function space, recovery of operator, error bounds of recovery.

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Received August 09, 2018
Revised November 06, 2018
Accepted November 12, 2018

Funding Agency: This work was supported by the Ministry of Education and Science of the Republic of Kazakhstan (grant no. AP05133257).

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Cite this article as:
D. B. Bazarkhanov. Linear recovery of pseudodifferential operators on classes of smooth functions on an m-dimensional torus. I, *Trudy Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 4, pp. 57–79 .