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OPTIMAL RECOVERY OF A FUNCTION ANALYTIC IN A HALF-PLANE FROM APPROXIMATELY GIVEN VALUES ON A PART OF THE STRAIGHT-LINE BOUNDARY

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Let $\mathcal{H}^p(\Pi_+,\phi)$ be the class of functions analytic in the upper half-plane Π_+ and belonging to the universal Hardy class N_* with boundary values from $L^p_\phi(\mathbb{R})$ with a weight ϕ, and let $Q^p(\Pi_+,\mathbb{I},\phi)$ be the class of function $f\in\mathcal{H}^p(\Pi_+,\phi)$ such that $\|f\|_{L^p_\phi(\mathbb{R}\setminus\mathbb{I})}\le 1$, where $\mathbb{I}$ is a finite open interval or a half-line from $\mathbb{R}$ and $1\le p\le\infty$. On the class $Q^p(\Pi_+,\mathbb{I},\phi)$, we consider the problem of optimal recovery of the value of a function at a point $z_0\in\Pi_+$ from its approximately given limit boundary values on $\mathbb{I}$ in the norm $L^p_\phi(\mathbb{I})$ and the related problem of the best approximation of a functional by linear bounded functionals. Explicit solutions of these problems are written: an extremal function, optimal recovery method, and best approximation functional. On the class $Q^p(\Pi_+,\mathbb{R}_+,\psi)$, $\psi(z)=1/|z|$, we solve the problem of optimal recovery of a function on a ray $\gamma=\{z\,:\,\arg z=\varphi_0\}$ with respect to the norm $L^p_\psi(\gamma)$ from its approximately given limit boundary values on $\mathbb{R}_+$ in the norm $L^p_\psi(\mathbb{R}_+)$ and the related problem of the best approximation of an operator by linear bounded operators. For $f\in\mathcal{H}^p(\Pi_+,\psi)$, we obtain the exact inequality

$$\|f\|_{L^p_\psi(\gamma)}\le\|f\|^{\varphi_0/\pi}_{L^p_\psi(-\infty,0)}\,\|f\|^{1-\varphi_0/\pi}_{L^p_\psi(0,+\infty)}.$$

Keywords: optimal recovery of an operator, best approximation of an unbounded operator by bounded operators, analytic function.

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