BEST ONE-SIDED APPROXIMATION IN THE MEAN
OF THE CHARACTERISTIC FUNCTION OF AN INTERVAL
BY ALGEBRAIC POLYNOMIALS

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M. V. Deikalova, A. Yu. Torgashova. Best one-sided approximation in the mean of the characteristic function of an interval by algebraic polynomials.

Let v be a weight on $(-1, 1)$, i.e., a measurable integrable nonnegative function different from zero almost everywhere on $(-1, 1)$. Denote by $L^v(-1, 1)$ the space of real-valued functions f integrable with weight v on $(-1, 1)$ with the norm $\|f\| = \int_{-1}^{1} |f(x)| v(x)\, dx$. We consider the problems of the best one-sided approximation (from below and from above) in the space $L^v(-1, 1)$ of the characteristic function of an interval (a, b), $-1 < a < b < 1$, by the set of algebraic polynomials of degree not exceeding a given number. We solve the problems in the case when a and b are nodes of a positive quadrature formula under some conditions on its degree of precision as well as in the case of a symmetric interval $(-h, h)$, $0 < h < 1$, for an even weight v.

Keywords: one-sided approximation, characteristic function of an interval, algebraic polynomials.

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