

ON THE PROPERTIES OF COMPLETENESS TYPE OF SPACES
OF FIRST FUNCTIONAL CLASS LEBESGUE MAPPINGS

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In this paper we study the Baire property and the Choquet property for the space $K_1(X, Y)$ — first functional class Lebesgue mappings, where X is a Tychonoff space and $Y \in \{\mathbb{R}, [0, 1], \{0, 1\}\}$. It is proved that the space $B_1(X, [0, 1])$ — $[0, 1]$ -valued Baire mappings of the first class is a Choquet (Baire) space if and only if the space $K_1(X, \{0, 1\})$ is a Choquet (Baire) space. The obtained studies allow us to quite simply solve V. Tkachuk's question about the coincidence of pseudocompactness and pseudocompleteness of the space $C_p(X, [0, 1])$.

Keywords: Baire property, Choquet space, Baire functions, Lebesgue functional class mappings, function space.

MSC: 54C35, 54E52; Secondary 54C30, 54H05

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REFERENCES

1. Aleksandrov P.S., Pasynkov B.A. *Vvedeniye v teoriyu razmernosti* [Introduction to the dimensionality theory]. Moscow, Mir Publ., 1973, 575 p.
2. Oxtoby J.C. *Measure and category. A survey of the analogies between topological and measure spaces*. New York, Springer, 1971. doi: 10.1007/978-1-4615-9964-7. Translated to Russian under the title *Mera i kategoriya*, Moscow, Mir Publ, 1974, 158 p.
3. Lutzer D., McCoy R. Category in function spaces. I. *Pacific J. Math.*, 1980, vol. 90, no. 1, pp. 145–168. doi: 10.2140/pjm.1980.90.145
4. Pytkeev E.G. Baire property of spaces of continuous functions. *Math. Notes*, 1985, vol. 38, no. 5, pp. 908–915. <https://doi.org/10.1007/BF01157538>
5. Tkachuk V. Characterization of the Baire property in $C_p(X)$ by the properties of the space X . *Research papers in Topology–Maps and extensions of topological spaces (Ustinov)*, 1985, pp. 21–27.
6. Douwen E.K. van Collected papers. Vol. 1 / ed. J. van Mill. Amsterdam: North-Holland Publ. Co., 1994. 767 p. ISBN-10: 0444816259.
7. Banakh T., Gabrielyan S. Baire category properties of some Baire type function spaces. *Topol. Appl.*, 2020, vol. 272, art. no. 107078. <https://doi.org/10.1016/j.topol.2020.107078>
8. Osipov A.V. Baire property of space of Baire-one functions // *Eur. J. Math.* 2025. Vol. 11. Art. no. 8. 25 p. <https://doi.org/10.1007/s40879-024-00799-1>
9. Lukeš J., Malý J., Zajíček L. *Fine topology methods in real analysis and potential theory*. Lecture Notes in Mathematics Ser., vol. 1189. Berlin, Heidelberg, Springer, 1986, p. 476. <https://doi.org/10.1007/BFb0075894>
10. Sakai M. Two properties of $C_p(X)$ weaker than the Fréchet — Urysohn property. *Topol. Appl.*, 2006, vol. 153, no. 15, pp. 2795–2804. <https://doi.org/10.1016/j.topol.2005.11.012>
11. Kąkol J., Kurka O., Leiderman A. Some classes of topological spaces extending the class of Δ -spaces. *Proc. Amer. Math. Soc.*, 2024, vol. 152, no. 2, pp. 883–898. <https://doi.org/10.1090/proc/16661>
12. Osipov A.V. Baireness of the space of pointwise stabilizing functions of the first Baire class. *Topol. Appl.*, 2025, vol. 362, art. no. 109218, 5 p. <https://doi.org/10.1016/j.topol.2025.109218>
13. Karlova O., Mykhaylyuk V. On stable Baire classes. *Acta Math. Hungar.*, 2016, vol. 150, no. 1, pp. 36–48. <https://doi.org/10.1007/s10474-016-0636-8>
14. Osipov A.V., Pytkeev E.G. Baire property of spaces of $[0, 1]$ -valued continuous functions. *Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat*, 2023, vol. 117, no. 1, art. no. 38, 10 p. <https://doi.org/10.1007/s13398-022-01371-w>

15. Osipov A.V. On the Baire property of the space of Baire indicator functions. *Mat. Zam.*, 2025, vol. 118, no. 4, pp. 564–574 (in Russian). <https://www.mathnet.ru/eng/mzm14478>
16. Kuratowski K. *Topology*, vol. 1. NY, Acad. Press, 1966, 560 p. <https://doi.org/10.2307/3611898>. Translated to Russian under the title *Topologiya*, Moscow, Mir Publ., 1966, 594 p.
17. Tkachuk V.V. *A C_p -theory problem book. Topological and function spaces*. NY, Springer, 2011, 488 p. <https://doi.org/10.1007/978-1-4419-7442-6>
18. Oxtoby J.C. The Banach–Mazur game and Banach category theorem. In: *Contributions to the theory of games*, Vol. 3; *Ann. of Math. Stud.*, no. 39, NJ, Princeton, Princeton Univer. Press, 1957. P. 159–163.
19. Osipov A.V. The Δ_1 -property of X is equivalent to the Choquet property of $B_1(X)$. *Topol. Appl.*, 2025, vol. 370, no. art. 109395, 6 p. <https://doi.org/10.1016/j.topol.2025.109395>
20. Banach T., Hryniv O. Some Baire category properties of topological groups. *Visnyk Lviv. Univ. Ser. Mech.-Mat.*, 2018, vol. 86, pp. 71–76. <https://doi.org/10.48550/arXiv.1901.01420>
21. Karlova O. On α -embedded sets and extension of mappings. *Comment. Math. Univ. Carolin.*, 2013, vol. 54, no. 3, pp. 377–396. <https://doi.org/10.48550/arXiv.1407.6155>
22. Hrušák M., Tamariz-Mascarúa Á., Tkachenko M. *Pseudocompact topological spaces. A survey of classic and new results with open problems*. Developments in Mathematics Ser., vol. 55. Cham, Springer, 2018, 299 p. <https://doi.org/10.1007/978-3-319-91680-4>

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