

AN ACCELERATED FEJÉR-TYPE PROCESS FOR FINDING A NON-NEGATIVE SOLUTION TO A SYSTEM OF LINEAR ALGEBRAIC EQUATIONS**V. I. Erokhin, G. Sh. Tamasyan, N. A. Stepenko**

The paper is in line with research, that was founded and developed in the papers of I. I. Eremin, V. V. Vasin, L. D. Popov, E. A. Berdnikova, I. M. Sokolinskaya, A. V. Ershova, E. A. Nurminskii and others. The main result is a new version of the Fejér-type mapping constructed for finding a non-negative solution to a system of linear algebraic equations. This mapping combines the operation of orthogonal projection onto a linear space of solutions to a system of linear algebraic equations and the operation of projection onto a non-negative orthant, without using the traditional positive slice operation, but instead using an elementwise operation of calculating the absolute value. The global linear convergence of the obtained algorithm is proved and its asymptotic constant is estimated. Computational experiments demonstrate significantly faster convergence of the studied mapping compared to the mapping using the positive slice operation. A description of the algorithm, its theoretical justification and the results of computational experiments are presented.

Keywords: Fejér-type processes, systems of linear equations and inequalities.

MSC: 15A06, 52B12, 65B99, 65K05, 90C99

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