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ON SMOOTHNESS OF THE BOUNDARY OF A REACHABLE SET UNDER INTEGRAL CONTROL CONSTRAINTS

M. I. Gusev

The paper considers reachable sets at a given time of linear control systems with integral constraints on control in the form of a ball in the space L_p for $p > 1$. The reachable sets are convex compact sets in the finite-dimensional Euclidean space $\mathbb{R}^n$. For $p = 2$, it is known that the reachable set, under the controllability condition, is an ellipsoid in $\mathbb{R}^n$ whose boundary is a compact smooth manifold diffeomorphic to a sphere. In this paper we obtain sufficient conditions under which the boundary of the attainability set turns out to be a smooth manifold of dimension $n - 1$ for all $1 < p \leq 2$.

Keywords: control system, integral constraints, reachable set, nonlinear mapping, maximum principle.

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Mikhail Ivanovich Gusev, Dr. Phys.-Math. Sci., Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia,
e-mail: gmi@imm.uran.ru .

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