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ON THE HYERS–ULAM–RASSIAS STABILITY OF NONLINEAR DIFFERENTIAL EQUATIONS CONTAINING PRODUCTS OF DISCONTINUOUS AND GENERALIZED FUNCTIONS AND DELAYS

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The article considers the Hyers–Ulam–Rassias stability property for nonlinear systems of differential equations with a generalized effect on the right-hand side. Since the right-hand side of the systems under consideration is unbounded, the standard definition of the stability property under consideration cannot be used. The formalization of the Hyers–Ulam–Rassias stability concept for nonlinear systems of differential equations with delay and discontinuous trajectories is given. Sufficient conditions are obtained that ensure such stability for a nonlinear system of differential equations with delay and a generalized effect on the right-hand side.

Keywords: Hyers–Ulam–Rassias stability, generalized action, differential equations, discontinuous trajectories.

MSC: 34D20, 34K20

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