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REPRESENTATION OF UNARS BY SETS OF RESIDUES

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We find faithful representations of a finite unar (an algebra with one unary operation on a finite set) in some standard constructions. We prove that every finite unar can be faithfully represented by the residues modulo n with the operation $f(x) = x \cdot a \mod n$ for suitable n and a. Besides, for every integer $d \geq 2$, there exists a faithful representation of every finite unar by residues modulo n with the operation $f(x) = x^d \mod n$ for suitable n. Further, for any $d \geq 3$, every finite unar can be faithfully presented by invertible residues modulo n with the operation $f(x) = x^d \mod n$ for suitable n. (The later assertion is not true for $d = 2$).

Keywords: representations of unar.

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