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ISOORDERLY PERMUTABLE SUBGROUPS OF FINITE GROUPS

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Let A and B be subgroups of a finite group G. Then the subgroup A is called: *isoorderly permutable with* B if there is a subgroup C of G such that $|C| = |B|$ and $AC = CA$, *hereditarily isoorderly permutable with* B if A is isoorderly permutable with B in any subgroup of G containing A and B, *isoorderly permutable in* G if A is isoorderly permutable with every subgroup of G, and *hereditarily isoorderly permutable in* G if A is hereditarily isoorderly permutable with every subgroup of G. In this paper, the properties of isoorderly permutable subgroups are analyzed, and the structure of a finite group G all of whose minimal subgroups are hereditarily isoorderly permutable is studied.

Keywords: finite group, isoorderly permutable subgroup, hereditarily isoorderly permutable subgroup, minimal subgroup.

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