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DISTRIBUTIVE LATTICES WITH DIFFERENT ANNIHILATOR PROPERTIES

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Distributive lattices with annihilator properties are studied. Abstract characterizations of such lattices are obtained. In particular, new characterization theorems are proved for generalized Boolean lattices and Boolean lattices, as well as for generalized Stone lattices and Stone lattices. It is proved that point distributive lattices are isomorphic to point lattices of sets. Examples are given. Generalizations of the obtained results to one class of commutative multiplicatively idempotent semirings are presented.

Keywords: distributive lattice, ideal, annihilator, annihilator properties, Boolean lattice, Stone lattice, point lattice, semiring.

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