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ON INTERSECTIONS OF π-Hall SUBGROUPS OF SOME D_π-GROUPS

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We study D_π-groups with a unit solvable radical that do not have nontrivial normal π-subgroups in which all simple nonabelian factors of their subnormal series are simple sporadic groups. It is proved that in such groups, for any π-Hall subgroup H, there exists an element g such that $H \cap H^g = 1$. Thus, Question 20.123 (c) of the Kourovka Notebook is solved and, under the above conditions, a positive answer is given to Question 18.31.

Keywords: Hall subgroup, D_π-group.

REFERENCES

1. Zenkov V.I., Mazurov V.D. On the intersection of Sylow subgroups in finite groups. *Algebra and Logic*, 1996, vol. 35, no. 4, pp. 236–240. https://doi.org/10.1007/BF02367025
2. Zenkov V.I. The intersections of nilpotent subgroups in finite groups. *Fundam. Prikl. Mat.*, 1996, vol. 2, no. 1, pp. 1–92 (in Russian).
3. Vdovin E.P. Regular orbits of solvable linear p'-groups. *Sib. Elektron. Mat. Izv.*, 2007, vol. 4, pp. 345–360.
4. Dolfi S. Large orbits in coprime actions of solvable groups. *Trans. Amer. Math. Soc.*, 2008, vol. 360, no. 1, pp. 135–152. https://doi.org/10.1090/S0002-9947-07-04155-4
5. Vdovin E.P., Revin D.O. Theorems of Sylow type. *Russian Math. Surv.*, 2011, vol. 66, no. 5, pp. 829–870. https://doi.org/10.1070/RM2011v066n05ABEH004762
6. The Kourovka notebook. Unsolved problems in group theory / eds. V.D. Mazurov, E.I. Khukhro. 20th ed. [e-resource]. Linkoln, Novosibirsk, University of Lincoln, U.K.; Inst. Math. SO RAN, 2024, 274 p. Available at: https://kourovka-notebook.org/ .
7. Zenkov V.I. On intersections of π-Hall subgroups in finite D_π-groups. *Sib. Math. J.*, 2022, vol. 63, no. 4, pp. 720-–722. https://doi.org/10.1134/S0037446622040127
8. Revin D.O. The D_π-property in finite simple groups. *Algebra and Logic*, 2008, vol. 47, no. 3, pp. 210–227. https://doi.org/10.1007/s10469-008-9010-4
9. Revin D.O., Vdovin E.P. On the number of classes of conjugate Hall subgroups in finite simple groups. *J. Algebra*, 2010, vol. 324, no. 12, pp. 3614–3652. https://doi.org/10.1016/j.jalgebra.2010.09.014
10. Zenkov V.I. Intersections of nilpotent subgroups in finite groups with sporadic socle. *Algebra and Logic*, 2020, vol. 59, no. 4, pp. 313–321. https://doi.org/10.1007/s10469-020-09603-x
11. Conway J.H., Curtis R.T., Norton S.P., Parker R.A., Wilson R.A. *Atlas of finite groups.* Oxford, Clarendon Press, 1985, 252 p. ISBN: 978-0-19-853199-9.
12. Gorenstein D. *Finite simple groups. An introduction to their classification.* New York, Springer, 1982, 333 p. https://doi.org/10.1007/978-1-4684-8497-7 . Translated to Russian under the title *Konechnye prostye gruppy. Vvedenie v ikh klassifikatsiyu*, Moscow, Mir Publ., 1985, 352 p.
13. Gorenstein D., Lyons R. *The local structure of finite groups of characteristic* 2 *type.* Mem. Amer. Math. Soc., vol. 42, edt. 276, Providence, RI, Am. Math. Soc., 1983, 731 p. https://doi.org/10.1090/memo/0276
14. Kabanov V.V., Kondrat'ev A.S. *Silovskiye* 2*-podgruppy konechnykh grupp* (obzor) [Sylow 2-subgroups of finite groups (Review)]. Sverdlovsk: Institute of Mathematics and Mechanics of the USSR Academy of Sciences Publ., 1979, 144 p.
15. Gluck D. Trivial set-stabilizers in finite permutation groups. *Canad. J. Math.*, 1983, vol. 35, no. 1, pp. 59–67. https://doi.org/10.4153/CJM-1983-005-2

16. Belousov I.N. I: Intersections π-Hall D_π-subgroups in finite simple sporadic groups, II: Finding a Lower Bound for $Orb_p(G)$ using the Sylow p-subgroup center centralizer. Available at: https://github.com/BelousovIN/Intersection_Subgroup , In: The GAP Group, GAP – Groups, Algorithms, and Programming, Version 4.14.0; 2024. (https://www.gap-system.org).
17. Vdovin E.P., Manzaev N.Ch., Revin D.O. On the heritability of the Sylow π-theorem by subgroups. *Sb. Math.*, 2020, vol. 211, no. 3, pp. 309–335. https://doi.org/10.1070/SM9185
18. Halasi Z., Podoski K. Every coprime linear group admits a base of size two. *Trans. Amer. Math. Soc.*, 2016, vol. 368, no. 8, pp. 5857–5887. https://doi.org/10.1090/tran/6544
19. Feit W., Thompson J.G. Solvability of groups of odd order. *Pacific J. Math.*, 1963, vol. 13, no. 3, pp. 775–787. https://doi.org/10.2140/pjm.1963.13.775
20. Glauberman G. Factorizations in local subgroups of finite groups. In: *CBMS Regional Conf. Ser. Math.*, vol. 33, Providence, R.I., Amer. Math. Soc., 1977. https://doi.org/10.1090/cbms/033
21. Kargapolov M.I., Merzlyakov Yu.I. *Osnovy teorii grupp* [Fundamentals of group theory], Moscow, Nauka Publ., 1982, 240 p.
22. Supplementary Materials for the article I. N. Belousov, V. I. Zenkov. "On intersections of π-Hall subgroups of some D_π-groups". The online version contains supplementary material available at http://journal.imm.uran.ru/Suppl_inf_2025-v.31-1 .

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