

MSC: 20G15

DOI: 10.21538/0134-4889-2024-30-4-77-83

**FULL AND ELEMENTARY NETS OVER THE FIELD OF FRACTIONS
OF A RING WITH THE QR-PROPERTY****R. Yu. Dryaeva, V. A. Koibaev**

The set $\sigma = (\sigma_{ij})$, $1 \leq i, j \leq n$, of additive subgroups σ_{ij} of a field K is called a net (carpet) over K of order n if $\sigma_{ir}\sigma_{rj} \subseteq \sigma_{ij}$ for all values of the indices i, r , and j . A net considered without the diagonal is called an elementary net. Based on an elementary net σ , an elementary net subgroup $E(\sigma)$ is defined, which is generated by elementary transvections $t_{ij}(\alpha) = e + \alpha e_{ij}$. An elementary net σ is called closed if the subgroup $E(\sigma)$ does not contain new elementary transvections. Suppose that R is a Noetherian domain with the QR-property (i.e., any intermediate subring lying between R and its field of fractions K is a ring of fractions of the ring R with respect to a multiplicative system in R), $\sigma = (\sigma_{ij})$ is a complete (elementary) net of order $n \geq 2$ ($n \geq 3$, respectively) over K , and the additive subgroups σ_{ij} are nonzero R -modules. It is proved that, up to conjugation by a diagonal matrix, all σ_{ij} are (fractional) ideals of a fixed intermediate subring P , $R \subseteq P \subseteq K$, and the inclusions $\pi_{ij}\pi_{ji} \subseteq P$ and $\pi_{ij} \subseteq P \subseteq \pi_{ji}$ hold for all $i < j$. In particular, the elementary net σ is closed.

Keywords: general and special linear groups, full and elementary nets (carpets) of additive subgroups, net subgroup.

REFERENCES

1. Gilmer R., Ohm J. Integral domains with quotient overrings. *Math. Ann.*, 1964, vol. 153, no. 2, pp. 97–103. doi: 10.1007/BF01361178
2. Borevich Z.I. Subgroups of linear groups rich in transvections. *J. Sov. Math.*, 1987, vol. 37, no. 2, pp. 928–934. doi: 10.1007/BF01089083
3. Levchuk V.M. Remark on a theorem of L. Dickson. *Algebra and Logic*, 1983, vol. 22, no. 4, pp. 306–316. doi: 10.1007/BF01979677
4. *The Kourovka notebook. Unsolved problems in group theory, 20th ed.*, eds. V.D. Mazurov, E.I. Khukhro, Novosibirsk: Inst. Math. SO RAN Publ., 2022, 269 p. Available at: <https://kourovka-notebook.org/>.
5. Koibaev V.A. Elementary nets in linear groups. *Tr. Instituta Matematiki i Mekhaniki UrO RAN*, 2011, vol. 17, no. 4, pp. 134–141 (in Russian).
6. Kuklina S.K., Likhacheva A.O., Nuzhin Ya.N. On closedness of carpets of Lie type over commutative rings. *Tr. Instituta Matematiki i Mekhaniki UrO RAN*, 2015, vol. 21, no. 3, pp. 192–196 (in Russian).
7. Atiyah M.F., Macdonald I.G. *Introduction to commutative algebra*. Reading, Mass., Addison—Wesley Publ. Co, 1969, 128 p. Translated to Russian under the title *Vvedeniye v kommutativnuyu algebru*. Moscow, Mir Publ., 1972, 160 p.
8. Dryaeva R.Y., Koibaev V.A., Nuzhin Ya.N. Full and elementary nets over the quotient field of a principal ideal ring. *J. Math. Sci.*, 2018, vol. 234, no. 2, pp. 141–147. doi: 10.1007/s10958-018-3990-y
9. Bourbaki N. *Algèbre commutative*. Paris, Hermann, 1961, 212 p. Translated to Russian under the title *Kommutativnaya Algebra*. Moscow, Mir Publ., 1971, 707 p.

Received January 23, 2024

Revised August 24, 2024

Accepted September 2, 2024

Funding Agency: The work was performed with the financial support of the Ministry of Science and Higher Education of the Russian Federation (Agreement number 075-02-2024-1447).

Roksana Yurievna Dryaeva, North-Ossetian State University named after K. L. Khetagurov, Vladikavkaz, 362025 Russia, e-mail: dryaeva-roksana@mail.ru .

Vladimir Amurkhanovich Koibaev, Dr. Phys.-Math. Sci., Prof., North-Ossetian State University named after K. L. Khetagurov, Vladikavkaz, 362025 Russia; Southern Mathematical Institute VSC RAS, Vladikavkaz, 362025 Russia, e-mail: koibaev-K1@yandex.ru .

Cite this article as: R. Yu. Dryaeva, V. A. Koibaev. Full and elementary nets over the field of fractions of a ring with the QR-property. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2024, vol. 30, no. 4, pp. 77–83 .