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## ON THE INFLUENCE OF THE $\mathfrak{F}$ -HYPERCENTER ON THE STRUCTURE OF FINITE MULTIFACTORIZED GROUPS

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The structure of a finite group having  $t$  (normal, abnormal) subgroups with pairwise mutually coprime indices and  $\mathfrak{F}$ -hypercentrality constraints on pairwise intersections of these subgroups, where  $\mathfrak{F}$  is a saturated formation, is described. As a consequence, new characterizations of  $\Sigma_t$ -closed, weakly  $R_t$ -closed, and  $N_0$ -closed formations of finite groups are established. The obtained results make a significant contribution to the solution of the question posed by W. Guo and A.N. Skiba in 2011.

Keywords: finite group, product of groups, saturated formation,  $\mathfrak{F}$ -hypercenter,  $\mathfrak{F}$ -central chief factor.

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