

MSC: 42C10, 47A58

DOI: 10.21538/0134-4889-2024-30-4-301-308

## $\mathcal{K}$ -FUNCTIONALS AND EXACT VALUES OF $n$ -WIDTHS FOR SOME CLASSES OF FUNCTIONS IN THE HARDY SPACE

M. Sh. Shabozov, R. A. Karimzoda

In this work, we obtain exact Jackson–Stechkin type inequalities in the Hardy space  $H_{q,\rho}$  ( $1 \leq q \leq \infty$ ,  $0 < \rho \leq R$ ), in which the values of the best polynomial approximations are estimated from above in terms of the  $\mathcal{K}$ -functionals of the  $r$ th derivatives. For function classes defined by the mentioned characteristics, exact values of Bernstein and Kolmogorov  $n$ -widths in the space  $H_{q,\rho}$  are calculated.

Keywords: Jackson–Stechkin type inequality, best polynomial approximation,  $\mathcal{K}$ -functional,  $n$ -widths.

### REFERENCES

1. Shabozov M.Sh. On the best simultaneous approximation in the Bergman space  $B_2$ . *Math. Notes*, 2023, vol. 114, iss. 3, pp. 377–386. doi: 10.1134/S0001434623090080
2. Shabozov M.Sh., Saidusainov M.S. Mean-square approximation of functions of a complex variable by Fourier sums in orthogonal systems. *Trudy Inst. Mat. Mekh. UrO RAN*, 2019, vol. 25, no. 2, pp. 258–272 (in Russian). doi: 10.21538/0134-4889-2019-25-2-258-272
3. Shabozov M.Sh. On the best simultaneous approximation of functions in the Hardy space. *Trudy Inst. Mat. Mekh. UrO RAN*, 2023, vol. 29, no. 4, pp. 283–291 (in Russian). doi: 10.21538/0134-4889-2023-29-4-283-291
4. Smirnov V.I., Lebedev N.A. *Konstruktivnaya teoriya funktsiy kompleksnogo peremennogo* [Constructive function theory complex variable]. Moscow, Nauka Publ., 1964, 438 p.
5. Pinkus A.  *$n$ -widths by approximation theory*. Berlin, Heidelberg, Springer, 1985, 294 p. doi: 10.1007/978-3-642-69894-1
6. Bergh I., Löfström I. *Interpolation spaces: an introduction*. Berlin, Springer, Softcover reprint of the original 1st ed. 1976 edition, 2011, 217 p. Translated to Russian under the title *Interpolyatsionnyye prostranstva: vvedeniye*, Moscow, Mir Publ., 1980, 264 p.
7. Vakarchuk S.B.  $K$ -functionals and exact values of  $n$ -widths for several classes from  $L_2$ . *Math. Notes*, 1999, vol. 66, no. 4, pp. 404–408. doi: 10.1007/BF02679087
8. Vakarchuk S.B. Mean approximation of functions on the real axis by algebraic polynomials with Chebyshev–Hermite weight and widths of function classes. *Math. Notes*, 2014, vol. 95, no. 5, pp. 599–614. doi: 10.1134/S0001434614050046
9. Shabozov M.Sh., Usupov G.A., Zargarov J.J. On the best simultaneous polynomial approximation of functions and their derivatives in Hardy spaces. *Trudy Inst. Mat. Mekh. UrO RAN*, 2021, vol. 27, no. 4, pp. 239–254 (in Russian). doi: 10.21538/0134-4889-2021-27-4-239-254
10. Shabozov M.Sh., Shabozova A.A., Mirkalonova M.M. Estimation of the remainder Taylor series for some classes of analytic functions with sums Taylor in Hardy space. *Dokl. NAN Tadjikistana*, 2023, vol. 66, no. 5–6, pp. 274–282 (in Russian).
11. Tikhomirov V.M. *Nekotoryye voprosy teorii priblizheniy* [Some questions of approximation theory]. Moscow, MGU Publ., 1976, 304 p.
12. Shevchuk I.A. *Priblizheniye mnogochlenami i sledy nepreryvnykh na otrezke funktsiy* [Approximation by polynomials and traces of continuous on a segment of functions]. Kyiv, Naukova Dumka Publ., 1992, 224 p.

Received September 2, 2024

Revised November 12, 2024

Accepted November 18, 2024

*Mirgand Shabozovich Shabozov*, Dr. Phys.-Math. Sci., Prof., Tajik National University, Dushanbe, 734025 Tajikistan, e-mail: shabozov@mail.ru .

*Ravshan Azam Karimzoda*, National University, Dushanbe, 734025 Tajikistan, e-mail: ravshan.karimov.93@mail.ru .

Cite this article as: M. Sh. Shabozov, R. A. Karimzoda.  $\mathcal{K}$ -Functionals and exact values of  $n$ -widths for some classes of functions in the Hardy space. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2024, vol. 30, no. 4, pp. 301–308 .