

MSC: 42A16; 42C40, 41A05

DOI: 10.21538/0134-4889-2024-30-4-286-300

**INTEGER EXPANSION OF ELEMENTS FROM SPACES
OF NONINTEGRABLE FUNCTIONS
INTO FOURIER-TYPE SERIES IN SYSTEMS OF CONTRACTIONS
AND SHIFTS OF ONE FUNCTION**

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We present results on the expansion of elements of the spaces $L_p(0, 1)$, $0 < p < 1$, into systems of functions that are contractions and shifts of one function. New algorithms are designed for the expansion of functions into Fourier-type series in these systems with integer coefficients. The expansions produced by the proposed methods have the property of image compression, i.e., a large number of expansion coefficients are zeros. The results may be of interest to specialists in the transmission and processing of digital information and to other researchers who have a need to expand nonintegrable functions into series in systems of contractions and shifts of one function.

Keywords: functional systems of contractions and shifts of one function of the space $L_p(0, 1)$ with $0 < p < 1$, Fourier-type series with integer coefficients, digital processing, information transmission.

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Received August 21, 2024

Revised November 4, 2024

Accepted November 18, 2024

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Cite this article as: V.I. Filippov. Integer expansion of elements from spaces of nonintegrable functions into Fourier-type series in systems of contractions and shifts of one function. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2024, vol. 30, no. 4, pp. 286–300.