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ON THE RELATION BETWEEN α -SETS AND WEAKLY CONVEX SETS

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The known relation between α -sets and Vial weakly convex sets in Euclidean spaces of dimension greater than two is strengthened. Namely, in the formula describing the relationship between nonconvexity measures for α -sets and weakly convex sets, the double Chebyshev radius is replaced by the diameter of the set with a coefficient. However, in the two-dimensional space the corresponding estimate is expressed in terms of the set diameter without a coefficient and is more accurate. In this connection, the question of the possibility of further refinement of the estimate for the nonconvexity degree α in terms of the weak convexity parameter R and the set diameter in Euclidean spaces of dimension greater than two remains open.

Keywords: α -set, weakly convex set, generalized convex set, diameter of a set, Chebyshev radius, convex hull.

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