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OPTIMAL EXTRAPOLATION OF POLYNOMIALS GIVEN WITH ERROR

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The problem of optimal extrapolation of polynomials given with an error on a compact set is studied. Its relationship with Chebyshev's problem on a polynomial that least deviates from zero on a compact set is established. An exact solution to the problem of optimal extrapolation of polynomials is obtained for the case when the compact set is a lemniscate. An exact solution is written for the problem of extrapolation from the interval $[-1, 1]$ to the real line.

Keywords: optimal extrapolation of polynomials, optimal recovery of functionals, Chebyshev polynomial of a compact set.

REFERENCES

1. Osipenko K.Yu. *Vvedeniye v teoriyu optimal'nogo vosstanovleniya* [Introduction to the theory of optimal recovery]. St. Petersburg, Lan', 2022, 388 p. ISBN: 978-5-507-44358-1.
2. Arestov V.V. Approximation of unbounded operators by bounded operators and related extremal problems. *Russian Math. Surv.*, 1996, vol. 51, iss. 6, pp. 1093–1126. doi: 10.1070/RM1996v051n06ABEH003001
3. Walsh J.L. *Interpolation and approximation by rational functions in the complex domain*. Rhode Island, Amer. Math. Soc., 1960, 405 p. ISBN: 9780821810200.
4. Chebyshev P.L. Theory of the mechanisms known as parallelograms. In: Chebyshev P.L. *Collected works. Vol. II. Mathematical analysis*. Moscow, Leningrad, Acad. Sci. USSR, 1947, pp. 23–51 (in Russian).
5. Faber G. Über Tschebyscheffsche polynome. *J. reine und angew. Math.*, 1920, vol. 150, pp. 79–106. doi: 10.1515/crll.1920.150.79
6. Milovanović G.V., Mitrinović D.S., Rassias Th.M. *Topics in polynomials: extremal problems, inequalities, zeros*. Singapore, World Sci. Publ. Comp., 1994, 821 p. ISBN: 981-02-0499-X.
7. Fischer B. Chebyshev polynomials for disjoint compact sets. *Constr. Approx.*, 1992, vol. 8, no. 3, pp. 309–329. doi: 10.1007/BF01279022
8. Peherstorfer F. Minimal polynomials for the compact sets of the complex plane. *Constr. Approx.*, 1996, vol. 12, no. 4, pp. 481–488. doi: 10.1007/BF02437504
9. Borodin P.A. On a condition for a polynomial that is sufficient for its norm to be minimal on a given compactum. *Vestnik Moskov. Univ. Ser. 1. Mat. Mekh.*, 2006, no. 4, pp. 14–18 (in Russian).
10. Pestovskaya A.E. Polynomials least deviating from zero with a constraint on the location of roots. *Tr. In-ta Matematiki i Mekhaniki UrO RAN*, 2022, vol. 28, no. 3, pp. 166–175 (in Russian). doi: 10.21538/0134-4889-2022-28-3-166-175
11. Suetin P.K. *Klassicheskiye ortogonal'nyye mnogochleny* [Classical orthogonal polynomials]. Moscow, Nauka Publ., 1979, 416 p.
12. Kochurov A.S., Tikhomirov V.M. On extrapolation of polynomials with real coefficients to the complex plane. *Math. Notes*, 2019, vol. 106, iss. 4, pp. 572–576. doi: 10.1134/S0001434619090256
13. Shabat B.V. *Vvedeniye v kompleksnyy analiz. Ch. 1. Funktsii odnogo peremennogo* [Introduction to complex analysis. Part 1. Functions of one variable]. St. Petersburg, Lan' Publ., 2004, 336 p.
14. Goluzin G.M. *Geometric theory of functions of a complex variable*. Translations of mathematical monographs, vol. 26. Providence, R.I., American Math. Soc., 1969, 676 p. doi: 10.1090/mmono/026. Original Russian text published in Goluzin G.M. *Geometricheskaya teoriya funktsii kompleksnogo peremennogo: Uchebnoe posobie*. Moscow, Leningrad, Nauka GITTL Publ., 1952, 628 p.

15. Akopyan R.R. Optimal recovery of analytic functions from boundary values specified with error. *Math. Notes*, 2016, vol. 99, iss. 2, pp. 177–182. doi: 10.1134/S000143461601020X

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