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ORDINAL SUMS OF RECTANGULAR SEMIGROUPS WITH OUTERPLANAR CAYLEY GRAPHS AND THEIR GENERALIZATIONS**D. V. Solomatin**

There are several equivalent definitions of the class of semigroups which we call rectangular semigroups. We will use the term rectangular semigroup to denote direct products of singular semigroups. A semigroup is called singular if it is a left zero semigroup or a right zero semigroup. A characteristic property of ordinal sums of rectangular semigroups with planar Cayley graphs is known. This article presents the characteristic properties of ordinal sums of rectangular semigroups with outerplanar Cayley graphs. In addition, the possibilities of their generalization to generalized outerplanar Cayley graphs of semigroups in the same class are analyzed. Namely, a necessary and sufficient condition for the existence of an outerplane embedding in the plane or generalized outerplane embedding in the plane of the Cayley graphs of the ordinal sums of rectangular semigroups is proved. The case when the Cayley graphs of such semigroups turn out to be generalized outerplanar, but not outerplanar, is considered in detail. The paper considers generalized outerplanar graphs characterized by Jiří Sedláček.

Keywords: outerplanar graph, semigroups with outerplanar Cayley graphs, generalized outerplanar graphs, Sedláček graphs, semigroups with planar Cayley graphs.

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