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**ONE EXAMPLE OF A CONTINUOUS NOWHERE DIFFERENTIABLE
FUNCTION WHOSE MODULUS OF CONTINUITY
DOES NOT EXCEED A GIVEN ONE**

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There exist positive numbers C and c such that, for an arbitrary concave up function $\omega(t)$ of the modulus of continuity type with $\omega(t)/t \rightarrow +\infty$ as $t \rightarrow +0$, one can construct an example of a continuous nowhere differentiable Weierstrass-type function $W_\omega(x)$ satisfying the following conditions:

1°. The modulus of continuity of $W_\omega(x)$ does not exceed $C\omega(t)$.

2°. For each point x_0 , there exists a sequence $\{x_n\}$ convergent to x_0 , such that $|W_\omega(x_n) - W_\omega(x_0)| > c\omega(|x_n - x_0|)$ for each n .

3°. At each point x_0 , the derivative numbers of $W_\omega(x)$ take all values from the interval $[-\infty; +\infty]$.

Keywords: modulus of continuity, nowhere differentiable continuous function, derivative numbers, Weierstrass-type nowhere differentiable continuous function.

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