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INEQUALITY OF DIFFERENT METRICS FOR DISCRETE LUXEMBURG NORMS IN FINITE-DIMENSIONAL SPACES

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An exact inequality of different metrics is obtained for discrete Luxemburg norms in a finite-dimensional space. As a consequence, using this inequality, an inequality of different metrics is proved for Luxemburg norms on functions for which there is an upper bound for the norm of a derivative in terms of the norm of the function, and an alternative proof is presented for S.M. Nikol'skii's inequality of different metrics for norms of a trigonometric polynomial in Orlicz spaces.

Keywords: inequality of different metrics, discrete Luxemburg norm, trigonometric polynomial, Orlicz space.

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