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## ON NORMAL DEDUCTION PROOFS IN INTUITIONISTIC NATURAL PREDICATE CALCULUS

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The theory constructed in the article includes the formulation of two natural first-order predicate calculi: intuitionistic and minimal. For these two types of logic, the article defines a new notion of normal natural deduction. A theorem on the normalization of natural deductions in these calculi is proved.

Keywords: system of natural deduction, intuitionistic natural predicate calculus, normal natural deduction, automated theorem proving, search for a natural logical proof.

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