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LATTICE CHARACTERIZATIONS OF p-SOLUBLE AND p-SUPERSOLUBLE FINITE GROUPS

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Let G be a finite group, and let $\mathcal{L}(G)$ be the lattice of all subgroups of G. A subgroup M of G is called *modular* in G if M is a modular element (in the Kurosh sense) of the lattice $\mathcal{L}(G)$, i.e., if (1) $\langle X, M \cap Z\rangle = \langle X, M\rangle \cap Z$ for all $X \leq G, Z \leq G$ such that $X \leq Z$, and (2) $\langle M, Y \cap Z\rangle = \langle M, Y\rangle \cap Z$ for all $Y \leq G, Z \leq G$ such that $M \leq Z$. If A is a subgroup of G, then A_{mG} is the subgroup of A generated by all its subgroups that are modular in G. We say that a subgroup A is *N-modular* in G ($N \leq G$) if, for some modular subgroup T of G containing A, N *avoids the pair* (T, A_{mG}), i.e. $N \cap T = N \cap A_{mG}$. Using these notions, we give new characterizations of p-soluble and p-supersoluble finite groups.

Keywords: finite group, p-soluble group, p-supersoluble group, modular subgroup, N-modular subgroup.

REFERENCES

1. Schmidt R. *Subgroup Lattices of Groups*, Berlin, Walter de Gruyter, 1994, 564 p.
2. Schmidt R. Endliche Gruppen mit vielen modularen Untergruppen. *Abhandl. Math. Semin. Univ. Hamburg*, 1969, vol. 34, iss. 1-2, pp. 115–125.
3. Skiba A.N. On sublattices of the subgroup lattice defined byformation Fitting sets. *J. Algebra*, 2020, vol. 550, pp. 69–85.
4. Huppert B. *Endliche Gruppen I.* Berlin; Heidelberg: Springer-Verlag, 1967, 796 p.

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